Prof. Rüdiger Frey
Institute for Statistics and Mathmatics
WU Vienna
ruediger.frey@wu.ac.at

Master QFin, CTFI
Minitest 2 Tue 30.5.2016, 9.15

Hints

- This is a closed-book exam; no pocket calculators etc.
- Please mark every sheet with your name and Mat.Nr.
- Good luck !

1. Stopping times. Consider a probability space $(\Omega, \mathcal{F}, P)$ with filtration $\{\mathcal{F}_t\}$.

a) (2 points) Give the definition of a stopping time.

b) (2 points) Consider two stopping times τ_1 and τ_2. Show that $\tau_* = \min\{\tau_1, \tau_2\}$ is a stopping time.

2. Brownian motion and martingales (2 points.) Let W_t, $t \geq 0$, be standard Brownian motion and define $\mathcal{F}_t := \sigma(W_s, s \leq t)$. Show that $W_t^2 - t$ is a martingale with respect to the filtration $\{\mathcal{F}_t\}$.

3. Application of the Ito formula (3 points) Let B be Brownian motion. Use Ito's formula to compute the semimartingale decomposition of $X_t = B_t^3$ and compute $[X]_t$

4. Application of the Ito formula (4 points) Consider a local martingale M with continuous trajectories. Let

$$Z_t = \exp\left(M_t - \frac{1}{2}[M]_t\right)$$

Use the Ito formula to show that Z satisfies the equation $Z_t = Z_0 + \int_0^t Z_s dM_s$. Is Z a local martingale? Compute Z for the case where $M_t = \sigma B_t$ for a standard Brownian motion B.